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From Recognition to Logical Deduction: Assessing the van Hiele Levels of Geometric Thinking Among Students of Diverse Disciplines

Markus Palobo[1*], Nasruddin[2], Yonarlianto Tembang[1], Oswaldus Dadi[1]

[1]Department of Mathematics Education, Musamus University
[2]Department of Mathematics Education, Universitas Sembilanbelas November Kolaka
Email Address: markuspalobo@unmus.ac.id
*Corresponding author

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***Abstract*:**
This study investigates the geometric thinking levels of university students using the van Hiele model as an analytical framework. A total of 85 students from four academic programs—Mathematics Education, Primary School Teacher Education, Agricultural Science, and Informatics Engineering—at Universitas Musamus participated in the study. The Van Hiele Geometry Test (VHGT), consisting of 25 multiple-choice items across five cognitive levels, was administered. Data were analyzed using descriptive statistical methods, including percentage distribution and level classification based on students' mastery of at least three out of five items at each van Hiele level. In addition, an item-based diagnostic analysis was conducted to identify patterns of misconceptions in students' responses. Comparative analysis across academic programs and gender was also performed to examine subgroup differences. The results revealed that most students were operating at the lower levels of geometric thinking: Level 0 (Pre-recognition), Level 1 (Visualization), and Level 2 (Analysis). Only two students reached Level 3 (Informal Deduction), and none achieved higher levels. Diagnostic findings indicated persistent misconceptions, particularly reliance on visual prototypes and difficulties in property-based reasoning. These findings highlight the need for instructional strategies aligned with students' cognitive levels to promote higher-order geometric reasoning. This study contributes to the literature by providing insights into cross-disciplinary differences in geometric thinking at the tertiary level.

Keywords: *van Hiele levels, geometric thinking, university student, informal deduction, diagnostic analysis*

Introduction

Geometric thinking is a foundational aspect of mathematical cognition, supporting learners in transitioning from visual recognition to abstract reasoning. It cultivates essential competencies such as spatial reasoning, conceptual abstraction, and logical argumentation, which are crucial not only in mathematics learning but also

in preparing prospective teachers for geometry instruction (Margaretha, 2025; Nopriana et al., 2023). However, evidence from multiple studies shows that many students remain at lower levels of geometric thinking, particularly when tasks require analysis or informal deduction (Sert Celik & Kaleli Yilmaz, 2022; Yanti et al., 2025).

The van Hiele theory of geometric reasoning offers a structured framework detailing five distinct levels through which students progress in their understanding of geometry. These levels are: Visualization (Level 0), Analysis (Level 1), Informal Deduction (Level 2), Deduction (Level 3), and Rigor (Level 4), as established by van Hiele in 1986 (Uyen et al., 2021). The fundamental premise of this theory is that students do not transition between these levels naturally as they age; instead, advancement requires purposeful and targeted instruction that corresponds to each specific level of understanding (Nur & Nurvitasari, 2017). This notion is supported by empirical findings, indicating that educators need to adapt teaching strategies so that they are aligned with the van Hiele model to facilitate effective learning outcomes (Aldiabat & Yew, 2024; Armah & Kissi, 2019).

A growing body of research has demonstrated the applicability of the van Hiele model in various educational contexts, particularly in improving students' geometric reasoning through structured instructional phases and the integration of metacognitive strategies. For instance, studies have shown that the use of van Hiele-based instruction and dynamic tools such as GeoGebra can significantly enhance students' ability to progress beyond visualization toward analytical and deductive thinking. However, most of these studies have focused primarily on school-level learners or mathematics education majors, leaving limited understanding of how the model applies to students from non-mathematics disciplines at the tertiary level. Recent studies have emphasized the effectiveness of integrating the van Hiele phases with metacognitive strategies (Naufal, Abdullah, Osman, Abu, Ihsan, et al., 2021) or digital tools like Geogebra (Gebremeskel et al., 2025) to enhance geometric understanding. Nevertheless, even pre-service teachers who are expected to possess stronger conceptual knowledge often operate at the visualization or analysis levels (Mbatha & Bansilal, 2023; Nopriana et al., 2023), reflecting inadequate exposure to structured deductive reasoning.

Most van Hiele-related research has predominantly focused on school-age learners and mathematics education majors. For example, several studies have examined geometric thinking levels among secondary school students and pre-service mathematics teachers, consistently reporting that many learners remain at the visualization or analysis levels. However, limited research has explored geometric thinking among students from non-mathematics disciplines at the tertiary level, indicating a gap in understanding how these cognitive processes develop across diverse academic backgrounds. Although some studies have also examined gender differences, these findings remain inconclusive and are not the primary focus of the present study. Few studies have assessed students from non-mathematics disciplines,

such as agricultural science, informatics engineering, or primary education disciplines where geometry is taught peripherally. This is a critical oversight, given the current emphasis on STEM integration and interdisciplinary competence, where spatial reasoning plays a vital role (Arlianti et al., 2025; Papademetri-Kachrimani, 2012). Furthermore, gender-based patterns in geometric cognition remain mixed, with some studies noting higher female performance in visual and logical reasoning (Tan et al., 2015), while others report negligible differences (Sert Celik & Kaleli Yilmaz, 2022).

The exploration of geometric reasoning among tertiary students from diverse disciplines is a pressing concern, particularly in regions where the curriculum emphasizes geometry less in non-mathematics programs. This lack of emphasis potentially hampers students' cognitive development and their ability to engage in higher-order thinking skills (HOTS). It is crucial to understand students' current cognitive levels concerning geometric reasoning so that pedagogical strategies can be effectively aligned with their needs, thereby preventing stagnation in their educational progression (Spelt et al., 2009).

This study responds to that need by investigating geometric thinking levels among students from four academic programs—Mathematics Education, Primary Education, Agricultural Science, and Informatics Engineering—at Universitas Musamus, Indonesia. This study aims to investigate students' levels of geometric thinking as conceptualized in the van Hiele model. More specifically, the research seeks to explore several key questions. First, what are the geometric thinking levels of university students from various academic programs namely Mathematics Education, Primary Education, Agricultural Science, and Informatics Engineering when assessed using the van Hiele model? The study also asks, are there differences in geometric thinking levels among students across these academic programs and between male and female students? In addition, it examines a third question: what types of misconceptions emerge from students' responses to the items in the Van Hiele Geometry Test (VHGT)?

By offering a detailed profile of students' geometric reasoning abilities, this study contributes to the growing literature on van Hiele-based assessment and provides actionable insights for improving geometry instruction in teacher education and STEM-related programs.

Research Methods

Research Design

This study employed a quantitative descriptive design with a supplementary item-based diagnostic analysis, focusing on the geometric thinking levels of university

students as conceptualized by the van Hiele model. The aim of this study was to (1) identify the distribution of students across van Hiele levels using percentage-based descriptive analysis, (2) compare patterns of geometric thinking across academic programs and gender through subgroup distribution analysis, and (3) analyze students' responses to higher-order items (Levels 2–3) using item-based diagnostic analysis to identify common misconceptions.

Participants

The participants in this study were first-year undergraduate students enrolled in four academic programs at Universitas Musamus: Mathematics Education, Primary Education, Agricultural Science, and Informatics Engineering. A total of 85 students took part in the research, consisting of 30 male and 55 female students. These participants were selected using purposive sampling to ensure the inclusion of a diverse academic background relevant to the study's objectives.

The decision to focus on first-year students was based on several pedagogical and research-based considerations. First, students in their initial year of university study are often transitioning from secondary to tertiary education, bringing with them pre-existing conceptions and misconceptions about geometry developed during their school years. This makes them ideal candidates for examining the baseline levels of geometric thinking prior to formal exposure to advanced mathematics or discipline-specific pedagogical content.

Second, because most general education or foundational mathematics courses are taken during the first year, this stage represents a critical cognitive window for assessing and potentially correcting misunderstandings before they become entrenched. By identifying the students' van Hiele levels early on, educational interventions can be more effectively designed to support the development of higher-order geometric reasoning.

Third, students from non-mathematics fields such as Agricultural Science and Informatics Engineering typically have limited formal instruction in geometry at the university level. Including them in the study allows for cross-disciplinary comparisons that are essential for evaluating the broader implications of geometric thinking beyond the field of mathematics education.

All students who participated had completed high school mathematics, including basic geometry, but had not yet taken any university-level geometry-specific courses at the time of data collection. Their participation was voluntary, and their diverse academic affiliations offered rich data for understanding how geometric thinking manifests across disciplines.

Participants were selected using purposive sampling based on their availability and willingness to participate in a diagnostic geometry test. The sample consisted of both male and female students across different academic years.

Table 1. Distribution of Participants by Academic Program and Gender

Program of Study	Male	Female	Total
Agricultural Science	5	6	11
Informatics Engineering	18	18	36
Primary School Teacher Edu.	5	9	14
Mathematics Education	2	22	24

This participant diversity allowed for meaningful comparison based on disciplinary background and gender.

Instrumentation

The primary instrument used in this study was the Van Hiele Geometry Test (VHGT), which consists of 25 multiple-choice items designed to assess geometric thinking levels from Level 1 (Visualization) to Level 5 (Rigor) based on the van Hiele model. Each item corresponds to a specific cognitive level and has been validated in prior international studies (Clements et al., 1997; Usiskin, 1982). Each item has a validated level designation and is grouped into five clusters:

Table 2. Cognitive Levels and Item Numbers in the Van Hiele Geometry Test (VHGT)

Level	Description	Item Numbers
1	Visualization	1–5
2	Analysis	6–10
3	Informal Deduction	11–15
4	Deduction/Formal Proof	16–20
5	Rigor (Axiomatic System)	21–25

The Indonesian version used in this study was adapted with permission from the original test, and pilot-tested for clarity with a small group of students not included in the main study.

Sample item example (Level 1):

"Which of the following figures is a square?" with diagrams requiring identification based on appearance.

Sample item example (Level 3):

"Statement 1: Shape F is a rectangle.

Statement 2: Shape F is a triangle.

Which of the following statements is true?" requiring logical reasoning about contradictory properties.

To determine students' geometric thinking levels, a mastery-based classification criterion was applied. Each van Hiele level in the Van Hiele Geometry Test (VHGT) consists of five items corresponding to a specific level of geometric thinking. A student was considered to have achieved a particular van Hiele level if they correctly answered at least three out of the five items at that level. The highest level at which the student met this mastery criterion was assigned as their geometric thinking level.

Students who did not meet the minimum criterion at Level 1 (Visualization) were classified as Level 0 (Pre-recognition). This classification procedure is consistent with established approaches in van Hiele-based assessment studies and ensures a systematic and objective identification of students' cognitive levels. In this study, the VHGT was adapted and translated into Indonesian, ensuring linguistic and contextual suitability for the student population. The instrument was adopted directly from the original version developed and standardized by (Usiskin, 1982) which has been widely used and empirically tested in various countries. As such, the instrument has established content validity and construct reliability based on decades of usage in research on geometric thinking (Armah & Kissi, 2019; Fujita & Jones, 2007).

To maintain its validity, the Indonesian version preserved the original structure, item sequencing, and cognitive level alignment. Furthermore, the translated instrument was pilot-tested with a small sample (n = 10) of university students who were not part of the main study, to confirm clarity and functionality of the adapted items. Due to its direct adoption from a validated instrument, and minimal modification in form or content, the VHGT used in this study is assumed to maintain its reliability. Previous studies reported internal consistency reliability coefficients ranging from 0.76 to 0.84, indicating a high level of measurement accuracy (Tsamir, Tirosh & Levenson, 2008; Fujita, 2012).

Data Collection Procedure

Data collection was carried out during the Odd Semester of the 2025/2026 academic year, integrated into scheduled class sessions across the four participating academic programs. Prior to data collection, students were informed about the objectives, procedures, and ethical considerations of the study. Both written and verbal informed consent were obtained before the administration of the test.

This sentence explains the test implementation procedure briefly and clearly. However, the author should add an explanation regarding the basis for determining the 35-minute duration to strengthen the validity of the measurement procedure. No calculators, measuring instruments, or digital tools were permitted, ensuring that responses reflected students' conceptual understanding rather than computational skill.

Following the test session, small-group classroom discussions were conducted to gather qualitative insights into students' reasoning. Selected students were invited to explain their choices particularly for items with high error rates. These discussions were audio-recorded (with consent) and later transcribed to support a deeper diagnostic analysis. This triangulation process enabled the researchers to connect students' answer patterns with their verbal reasoning, providing a richer understanding of misconceptions and cognitive processes across van Hiele levels.

All test responses were anonymized and coded before analysis. Data were then grouped by academic program, gender, and item performance to allow both quantitative profiling and qualitative interpretation of geometric thinking across the participant group.

Data Analysis

The data obtained from the Van Hiele Geometry Test (VHGT), which consisted of 25 multiple-choice items, were analyzed using both quantitative and qualitative approaches to provide a comprehensive understanding of students' geometric thinking levels. Each student's responses were first categorized according to the van Hiele levels Level 1 (Visualization), Level 2 (Analysis), Level 3 (Informal Deduction), Level 4 (Deduction), and Level 5 (Rigor) based on the established mapping of each item to its respective cognitive level.

Quantitatively, the number and percentage of correct responses for each item were tabulated to identify patterns in performance across the five van Hiele levels. This analysis was further disaggregated by academic program (Mathematics Education, Primary Education, Agricultural Science, and Informatics Engineering) and gender (male and female) to examine whether any significant trends or differences emerged. The distribution of scores across levels was used to construct students'

geometric thinking profiles, helping to reveal the general cognitive positioning of students within the van Hiele framework.

In addition to the statistical analysis, a qualitative diagnostic approach was used to analyze students' reasoning processes. Students' verbal explanations collected during classroom discussions were transcribed and analyzed using a thematic coding approach. The analysis involved three stages: (1) initial coding to identify key patterns in students' reasoning, (2) categorization of responses into recurring themes such as prototype dependence, property-based reasoning, and logical misconceptions, and (3) interpretation of these themes in relation to the van Hiele levels of geometric thinking. This process allowed for a systematic identification of common misconceptions and reasoning patterns underlying students' responses. Explanations collected during classroom discussions were transcribed and reviewed to identify common misconceptions and reasoning patterns. Students' verbal justifications were compared to their selected answers on the VHGT to validate whether their choices reflected true conceptual understanding or were influenced by visual biases or incomplete logic. This triangulated analysis allowed for a richer interpretation of students' thinking, particularly in explaining why certain items had high error rates and what types of reasoning led to incorrect answers.

Together, these two methods quantitative scoring and qualitative interpretation provided a robust foundation for evaluating not only the students' current geometric thinking levels but also the underlying factors affecting their reasoning abilities.

Ethical Considerations

This research involved human participants university students from diverse academic programs and was conducted in accordance with ethical research standards. Participation was entirely voluntary, with no coercion involved. Students were clearly informed about the purpose and nature of the study, and informed consent was obtained before data collection. Participants were assured that their data would remain confidential and anonymous, and they were free to withdraw at any point without any academic penalty. No psychological, physical, or academic risks were posed to the participants. Ethical approval for this study was obtained through internal coordination with the faculty's research unit at Universitas Musamus, ensuring that all institutional ethical guidelines were followed.

Results and Discussions

Overall Distribution of van Hiele Thinking Levels

The analysis revealed that the geometric thinking levels of the 85 participants were concentrated at the lower levels of the van Hiele model. As shown in Table 3, the

majority of students were classified at Level 0 (Pre-Recognition) and Level 1 (Visualization), with only a small number reaching Level 3 (Informal Deduction). No students attained Levels 4 or 5.

Table 3. Overall Distribution of Geometric Thinking Levels

Level	Male	Female	Total
0 (Pre-Recognition)	20	18	38
1 (Visualization)	5	28	33
2 (Analysis)	5	7	12
3 (Informal Deduction)	0	2	2
4 (Deduction)	0	0	0
5 (Rigor)	0	0	0

These findings confirm that the vast majority of students had not progressed beyond basic levels of geometric reasoning. The low number of students reaching Level 3 (only 2 females) suggests a significant gap in students' ability to reason informally about geometric relationships, consistent with prior studies (Mbatha & Bansilal, 2023).

Distribution by Academic Program

Table 4 presents the level distribution disaggregated by academic program. Students in the Mathematics Education program showed slightly higher overall levels of geometric thinking. However, students who reached Level 3 (Informal Deduction) were identified in both the Mathematics Education and Informatics Engineering programs. In contrast, other programs, particularly Agricultural Science and Primary Education, were dominated by Level 0 (Pre-Recognition) and Level 1.

Table 4. van Hiele Levels by Academic Program

Level	Agricultural	Informatics	Primary Edu	Math Edu	Total
0	6	15	9	8	38
1	5	14	3	11	33
2	0	6	2	4	12
3	0	1	0	1	2
4	0	0	0	0	0

Students in the Mathematics Education program demonstrated comparatively better performance, with a higher percentage of students reaching Level 2 and Level 3 compared to other programs. However, their progression beyond Level 2 remained

limited. This reinforces the concern that even students training to be mathematics educators may not possess the geometric reasoning necessary to facilitate higher-level learning.

Distribution by Gender

Table 5 shows the distribution of van Hiele levels by gender. Female students were more widely distributed across levels, with two females reaching Level 3, while no male participants did.

Table 5. van Hiele Levels by Gender

Level	Male	Female
0	20	18
1	5	28
2	5	7
3	0	2
4	0	0

These results indicate that male students were predominantly concentrated at Level 0, accounting for 20 out of 30 students (66.7%), whereas female students showed a more distributed performance across levels. Specifically, 37 out of 55 female students (67.3%) reached Levels 1–3, compared to only 10 out of 30 male students (33.3%). This suggests that female students tended to perform better at higher levels of geometric thinking. This pattern is consistent with findings in other contexts, where female students demonstrate more consistent visual and analytical reasoning, although overall levels remain low (Armah & Kissi, 2019).

Diagnostic Item-Based Analysis (Items 1–15)

A diagnostic analysis was conducted on Items 1–15 of the Van Hiele Geometry Test (VHGT), representing Levels 1–3 of geometric thinking. These items assess students' ability to recognize shapes (Level 1), analyze properties (Level 2), and apply informal logical reasoning (Level 3). The analysis revealed recurring patterns of misconceptions, which are discussed thematically based on van Hiele levels.

Level 1 (Visualization): Prototype Dependence and Visual Bias

At Level 1, students predominantly relied on visual appearance rather than formal definitions when identifying geometric shapes. Across Items 1–5, many students failed to recognize valid shapes presented in non-prototypical forms, such as elongated rectangles, rotated figures, or non-standard triangles. For example, in identifying

quadrilaterals (Item 1), a substantial number of students selected only the square and rejected the rectangle, as illustrated by the following response:

"*I chose L only because it looks like a square. K doesn't look like a quadrilateral—it's too long.*" (PGSD student)

Similarly, in identifying triangles (Item 2), students showed strong dependence on orientation-based prototypes:

"*I picked V because it's pointy like a triangle. The others are too wide*." (Agricultural student)

A comparable pattern appeared in Item 3, where rotated rectangles were frequently rejected:

"*S is a rectangle, but T doesn't count—it looks slanted*." (PGSD student)

These responses indicate that students classify shapes based on familiar visual prototypes (e.g., upright, symmetrical forms) rather than invariant properties such as number of sides or angle measures. This finding aligns with prior studies indicating that learners at the Visualization level rely heavily on perceptual features and struggle to generalize across varied representations (Fujita & Jones, 2007; Sinclair & Bruce, 2015). Furthermore, in Items 4 and 5, students exhibited overgeneralization and orientation bias. Some identified multiple "square-like" figures regardless of angle accuracy, while others failed to recognize parallelism in rotated shapes:

"*I didn't pick the tilted ones because they don't look parallel*." (Agricultural student)

Overall, these findings confirm that most students operate at a visual recognition stage, where geometric understanding is dominated by appearance rather than formal properties. This suggests limited readiness to progress toward analytical reasoning.

Level 2 (Analysis): Fragmented Property Understanding

At Level 2, students began to recognize geometric properties; however, their reasoning remained partial and fragmented. Across Items 6–10, students typically focused on a single attribute while ignoring other defining conditions. For instance, in Item 6, students identified squares based on equal side lengths but neglected perpendicularity:

"*I chose the option with equal lines—squares should have same lengths*." (Informatics student)

Similarly, in Item 7, students showed incomplete understanding of rectangle properties:

"I just know the sides are equal, not sure about diagonals." (PGSD student)

In Item 8, a common misconception involved overgeneralization of square properties to rhombuses:

"*A rhombus is like a square, so everything should be true*." (Agricultural student)

These responses indicate that students have begun to identify individual properties but have not yet developed the ability to coordinate multiple attributes simultaneously. This aligns with van Hiele Level 2 characteristics, where learners can describe properties but struggle to form integrated conceptual structures (Sinclair & Bruce, 2015). In addition, Item 9 revealed that students could recognize isolated properties (e.g., equal sides or angles) but failed to establish logical relationships between them:

"*I know the sides are the same, but I didn't think about angles*." (PGSD student)

Meanwhile, Item 10 highlighted difficulties in interpreting non-standard geometric representations, with some students resorting to guessing:

"*I didn't really get the picture, so I picked the most complete answer*." (Informatics student)

These findings suggest that students' analytical reasoning remains underdeveloped, particularly in coordinating multiple properties and interpreting complex visual information.

Level 3 (Informal Deduction): Weak Logical Reasoning

At Level 3, students were expected to demonstrate informal deductive reasoning, including understanding logical relationships, contradictions, and class inclusion. However, the results indicate that most students have not yet reached this level. In Item 11, some students accepted contradictory statements as simultaneously valid:

"*Maybe the shape is both a mix between a rectangle and triangle*." (PGSD student)

This reflects a lack of understanding of logical contradiction and mutual exclusivity in geometric definitions. Similarly, in Item 12, students struggled with conditional reasoning:

"*I picked what felt right—I don't really understand logic questions*." (Math student)

In Items 13–15, students continued to rely on visual features and demonstrated limited understanding of hierarchical relationships among shapes. For example, some rejected rotated rectangles or failed to recognize that squares are a subset of rectangles:

"*Only the straight ones look like rectangles. The others are tilted*." (Math student)

"*They're different shapes, so their properties must be different too*." (PGSD student)

Additionally, overgeneralization persisted in reasoning about diagonal properties:

"*Any shape with right angles must have same-length diagonals." (Informatics student)*

These responses indicate that students struggle with fundamental aspects of informal deduction, including class inclusion, logical implication, and property differentiation. This suggests that most participants remain below van Hiele Level 3.

To further synthesize these findings, Table 6 summarizes the dominant misconceptions across van Hiele levels along with representative student responses.

Table 6. Summary of Students' Misconceptions Across van Hiele Levels

Level	Cognitive Characteristics	Common Misconceptions	Representative Student Responses	Interpretation
Level 1 (Visualization)	Recognition based on visual appearance	Prototype dependence; rejection of non-standard shapes (rotated, elongated); orientation bias	"It doesn't look like a quadrilateral—it's too long." "I picked V because it's pointy like a triangle." "It doesn't count—it looks slanted."	Students rely on visual prototypes rather than defining properties, indicating perception-based reasoning typical of early geometric thinking (Fujita & Jones, 2007).
Level 2 (Analysis)	Identification of properties, but not fully integrated	Single-attribute reasoning; overgeneralization (e.g., square → rhombus); fragmented	"Squares should have same lengths." "I just know the sides are equal, not sure about	Students recognize individual properties but fail to coordinate multiple

Level	Cognitive Characteristics	Common Misconceptions	Representative Student Responses	Interpretation
		understanding of properties	diagonals." "A rhombus is like a square, so everything should be true."	attributes, reflecting incomplete analytical reasoning and weak conceptual structure (Sinclair & Bruce, 2015).
Level 3 (Informal Deduction)	Emerging logical reasoning, but inconsistent	Difficulty with contradiction, conditional reasoning, and class inclusion; overgeneralization of properties	"Maybe the shape is both a rectangle and triangle." "I don't really understand logic questions." "They're different shapes, so their properties must be different."	Students struggle with logical relationships and hierarchical classification, indicating limited development of informal deductive reasoning.

General Trends in Geometric Thinking

The results of this study indicate a dominant trend of low geometric reasoning among university students across all four academic programs. Most participants were concentrated at the lower levels of geometric thinking, with 38 out of 85 students (44.7%) at Level 0 (Pre-recognition) and 33 students (38.8%) at Level 1 (Visualization). A smaller proportion reached Level 2 (Analysis), accounting for 12 students (14.1%), while only two students (2.4%), both females, attained Level 3 (Informal Deduction). This distribution indicates that higher-order geometric reasoning remains underdeveloped among the participants. Although Level 3 students were female, this finding should be interpreted cautiously due to the small number of cases. These findings are consistent with previous research showing that learners often rely on superficial visual features and struggle to engage in property-based or logical reasoning tasks (Fujita & Jones, 2007; Mbatha & Bansilal, 2023; Sert Celik & Kaleli Yilmaz, 2022).

The van Hiele model posits that without explicit, structured instruction, students are unlikely to progress through the levels of geometric thinking. This theory is affirmed by the diagnostic data gathered in this study, which reveal that even students

in teacher education programs frequently default to prototype-based identification. For example, in Item 1, many students rejected rectangles as quadrilaterals, assuming that only squares fit the category. This misconception appears to stem from prototype-based reasoning, where students rely on familiar visual representations rather than formal definitions. Such reasoning is often influenced by prior learning experiences that emphasize typical examples (e.g., squares) while providing limited exposure to varied representations of quadrilaterals. As a result, students struggle to generalize the concept based on defining properties such as having four sides. This pattern of thinking is characteristic of students operating at van Hiele Level 0 (Visualization), where classification is primarily based on appearance rather than analytical understanding of properties. As one student explained, "I chose L only because it looks like a square. K doesn't look like a quadrilateral—it's too long," reflecting a Level 0 understanding.

Misconceptions and Cognitive Gaps

A detailed analysis of Items 1–15 uncovered consistent misconceptions and reasoning gaps. Many students demonstrated prototype dependence—selecting or rejecting figures based on how "typical" they looked (e.g., only choosing upright triangles or rectangles, rejecting rotated figures). This is evident in Item 3 and Item 13, where students failed to recognize rotated rectangles (Fujita & Jones, 2007). At Level 2, students struggled with property-based reasoning, particularly when identifying which properties apply consistently across classes of shapes. In Item 8, for instance, students overgeneralized properties of squares to rhombuses, assuming all rhombi have equal diagonals. Similarly, in Item 9, they were unaware of the inherent relationship between side and angle properties in isosceles triangles, revealing fragmented conceptual understanding (Sinclair & Bruce, 2015).

When presented with Level 3 items, students showed difficulty in applying informal deductive reasoning. For example, in Item 11, some students believed two contradictory statements could both be true. As one stated: "I thought both could be right, maybe it's a mixed shape." This reflects a lack of understanding of logical contradiction, a foundational concept in deductive reasoning (Papademetri-Kachrimani, 2012). For example, a geometric figure cannot simultaneously be classified as both a triangle (having three sides) and a rectangle (having four sides), as these definitions are mutually exclusive. Students' acceptance of such contradictory statements indicates difficulty in applying basic logical principles. This has important implications for geometry learning, as the ability to recognize contradictions is essential for developing deductive reasoning and constructing valid geometric arguments. Similarly, Item 12 revealed that students had trouble identifying conditional implications, echoing concerns from previous studies on logic gaps in geometry education (Armah & Kissi, 2019; Usiskin, 2021).

Disciplinary and Gender Differences

While Mathematics Education students performed slightly better overall, with one student reaching Level 3, the general level of geometric reasoning across all disciplines—including STEM and non-STEM was low. These findings echo prior studies showing that mathematics teacher candidates may lack the deductive skills necessary for effective geometry instruction (Mbatha & Bansilal, 2023; Nopriana et al., 2023). Female students were more likely to reach Level 2 and Level 3 than male students, which supports some prior research indicating stronger verbal and relational reasoning tendencies in female learners (Sert Celik & Kaleli Yilmaz, 2022; Tan et al., 2015). However, the small sample size and contextual specificity require cautious interpretation.

Conclusions and Suggestions

This study examined the geometric thinking levels of university students from four academic programs using the Van Hiele Geometry Test (VHGT). The findings indicate that most students remain at the lower levels of geometric reasoning, particularly at Level 0 (Pre-recognition) and Level 1 (Visualization), with only a small proportion reaching Level 2 (Analysis) and very few attaining Level 3 (Informal Deduction). No students demonstrated higher-level reasoning (Levels 4 and 5). The diagnostic analysis revealed persistent misconceptions, including reliance on visual prototypes, fragmented understanding of geometric properties, and difficulty in applying logical reasoning. These findings suggest a significant gap between expected and actual levels of geometric thinking, even among prospective mathematics teachers. Consistent with the van Hiele theory, these results confirm that progression in geometric thinking requires structured and level-appropriate instruction. Without targeted pedagogical interventions, students are likely to remain at superficial levels of understanding, limiting their ability to engage in formal reasoning and proof.

Based on these findings, several implications are proposed. First, geometry curricula at the university level should emphasize conceptual understanding and reasoning processes, rather than procedural knowledge alone. Second, instructional strategies should be aligned with students' van Hiele levels, incorporating visual, analytical, and deductive learning experiences. Third, additional support is needed for students from non-mathematics disciplines through the use of visual tools, hands-on activities, and contextual learning approaches. Fourth, teacher education programs must strengthen both content knowledge and pedagogical competence in teaching geometry conceptually. Finally, future research should involve larger and more diverse samples, as well as longitudinal or intervention-based designs, to better understand the development of geometric thinking and the effectiveness of instructional approaches.

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