

Didactic Obstacles in Trigonometric Ratios: Insights into Students' Mathematical Understanding

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Abstract:

This study examines the relationship between the knowledge prescribed in the school curriculum, the knowledge enacted by mathematics teachers, and the knowledge constructed by students in learning trigonometric ratios through the perspective of Didactic Transposition Theory. A qualitative intrinsic case study was conducted with one mathematics teacher and 22 tenth-grade students in an Indonesian senior high school. Data were collected from curriculum documents, teaching notes, classroom observations, interviews, and two tests designed to capture students' conceptual understanding. The data were analyzed using document analysis, content analysis, and descriptive analysis. The findings show that although students' understanding of trigonometric ratios improved during instruction, the enacted classroom practices mainly emphasized trigonometric ratios as fixed relationships among the sides of right triangles. Consequently, students tended to interpret sine, cosine, and tangent primarily as geometric ratios rather than as functions of angles. This emphasis limited students' ability to connect geometric reasoning with functional interpretations of trigonometric relationships. These results highlight the importance of strengthening instructional coherence in curriculum implementation so that classroom teaching better supports the development of both geometric and functional understanding of trigonometry.

Keywords: Trigonometric ratios; Didactic Transposition Theory; Learning obstacles; Right triangle trigonometry; Pedagogical content knowledge

Introduction

Trigonometry constitutes an essential component of the Indonesian senior high school mathematics curriculum, where students are introduced to fundamental concepts, applications, and trigonometric ratios such as sine, cosine, and tangent (Kemendikbud, 2016). In classroom practice, trigonometric knowledge is typically introduced through right-angled triangles, emphasizing ratio relationships among the



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sides. However, learning trigonometry involves more than procedural manipulation of ratios. It requires students to develop both instrumental and relational understanding, particularly the transition from interpreting trigonometry as static ratio relationships to understanding it as a coherent system of mathematical functions (Çekmez, 2020; Weber, 2005a; Yiğit Koyunkaya, 2016). Despite its importance, many students experience difficulties in constructing coherent meanings of trigonometric ratios. These difficulties often emerge from discrepancies between the formal mathematical definitions presented during instruction and students' internal conceptual representations (Solstad et al., 2025; Tall & Vinner, 1981; Vinner, 1983). Conceptual representations reflect the cognitive structures through which learners interpret and reason about mathematical ideas, and mismatches between instructional discourse and students' cognitive constructions may result in fragmented or superficial understanding (Jones & Mamona-Downs, 2008). Consequently, understanding how mathematical knowledge is interpreted and reconstructed by students has become a critical issue in trigonometry education.

Previous studies have documented a wide range of misconceptions and conceptual difficulties related to trigonometric ratios. Research has identified persistent challenges in interpreting angle concepts, ratio structures, symbolic representations, and mathematical relationships (Fikadu Aga, 2024; Hamzah et al., 2021; Juárez-Ruiz & Sliško, 2024; Scheiner & Bosch, 2023; Tole Fikadu Aga, 2024). Specific misconceptions include rigid interpretations of angle positioning (OZEN UNAL & URUN, 2021); errors in symbolic manipulation and equation usage (Çekmez, 2020); and confusion in identifying triangle sides relative to different angles (Mulyono & Hapizah, 2021). Other studies emphasize students' difficulties in information processing, conceptual application, and comparative reasoning (Nurmeidina & Rafidiyah, 2019). While these studies provide valuable insights into students' errors, most of the existing literature primarily focuses on identifying misconceptions rather than explaining how such conceptual difficulties emerge through instructional processes. In particular, limited attention has been given to examining how the transformation of mathematical knowledge across curriculum design, classroom instruction, and students' interpretations may contribute to these learning difficulties.

From a didactical perspective, students' difficulties in mathematics can be interpreted through the notion of learning obstacles. According to (Brousseau, 2002), learning obstacles may be classified into epistemological obstacles, ontogenetic obstacles, and didactical obstacles. Epistemological obstacles arise from the inherent complexity of mathematical concepts themselves, whereas ontogenetic obstacles relate to students' cognitive development and readiness to understand certain ideas. In contrast, didactical obstacles originate from the ways knowledge is organized, and presented in instructional settings, including the representations used, the sequencing of concepts, and the teaching approaches employed. Understanding these obstacles is important because instructional practices may shape how students construct meanings of mathematical concepts.

One theoretical framework that provides insight into the transformation of knowledge within educational systems is Didactic Transposition Theory (DTT) developed by (Chevallard, 1991). DTT conceptualizes knowledge not as a static entity but as an object that undergoes continuous transformation as it moves across different institutional contexts. Knowledge produced in scholarly domains is first transformed through external didactic transposition by the noosphere—including curriculum designers, policymakers, and educational institutions—before being reshaped through internal didactic transposition within classroom environments (Abels et al., 2019; Bosch & Gascón, 2006a). Through teachers’ instructional decisions, pedagogical practices, and representational choices, prescribed knowledge becomes taught knowledge, which students subsequently reconstruct as learned knowledge.

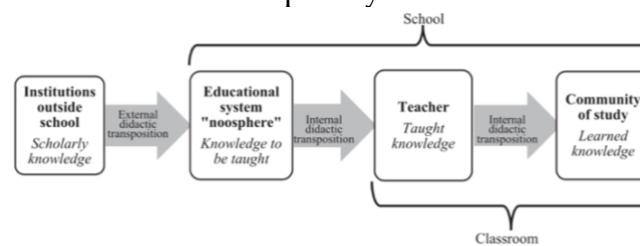


Figure 1. The didactic transposition process by Bosch & Gascón

Figure 1 illustrates the didactic transposition process as conceptualized by Bosch and Gascón, describing how knowledge is transformed through several stages before being learned by students in the classroom. In the complete model, the process begins with scholarly knowledge and is progressively transformed into knowledge suitable for teaching and learning. However, this study does not analyze the scholarly knowledge level because the focus of the research lies in the instructional processes occurring within the school context. Therefore, the analysis is limited to three levels: knowledge to be taught, taught knowledge, and learned knowledge. Knowledge to be taught refers to how trigonometric ratio concepts are structured and represented in curriculum documents and textbooks. Taught knowledge describes how teachers interpret and implement this content in classroom instruction. Meanwhile, learned knowledge reflects how students understand and reconstruct the concepts through their learning experiences. Focusing on these three levels allows for a more in-depth examination of how the transformation of knowledge within the teaching learning process may generate didactic obstacles in students’ understanding of trigonometric ratios.

Despite the relevance of Didactic Transposition Theory for examining the dynamics of mathematical knowledge in educational contexts, studies applying this perspective to trigonometric learning particularly in the Indonesian context remain limited. More importantly, little is known about how the process of didactic transposition may contribute to students’ tendency to interpret trigonometric concepts primarily as ratio relationships rather than as mathematical functions. Grounded in Didactic Transposition Theory, this study aims to investigate the relationships between the knowledge prescribed in the Indonesian mathematics curriculum, the

knowledge enacted by teachers, and students' conceptual understanding of trigonometric ratios. Specifically, the study examines how the didactic transposition process shapes students' interpretations of trigonometry, with particular attention to the observed tendency of students to remain confined to ratio-based reasoning rather than developing function-based understanding. By analyzing the transformation of knowledge across curriculum, instruction, and student cognition, this research seeks to provide a deeper explanation of the origins of conceptual obstacles in trigonometric learning.

Research Methods

This study employed an intrinsic case study design aimed at providing an in-depth understanding of a particular educational case and its components (Creswell & Poth, 2018). The research focused on examining the internal didactic transposition process within the framework of Didactic Transposition Theory (DTT), particularly the relationships among the knowledge to be taught, the taught knowledge, and the learned knowledge. A case study approach was adopted because the study investigated a contemporary educational phenomenon situated within its real-life classroom context (Yin, 2014). The study involved one female mathematics teacher and a Grade 10 class from a senior high school in Indonesia. The teacher volunteered to participate in the study and had five years of teaching experience, with an academic background in mathematics education. The class consisted of 22 Grade 10 students (12 females and 10 males). The class was selected because trigonometric ratios were being introduced for the first time at this level of the curriculum.

Data collection was conducted in four phases. The first phase examined the knowledge to be taught through curriculum analysis and a preliminary teacher interview. The interview explored instructional planning, including teaching resources, instructional tools, learning materials, and pedagogical strategies. The interview was audio-recorded and subsequently transcribed. Curriculum documents were analyzed to identify the intended learning objectives related to ratios and trigonometry (Kemendikbud, 2016). The second phase assessed students' prior knowledge using a Readiness Test (RT). The RT was developed based on curriculum objectives and was validated by a mathematics education expert. The validation focused on the clarity of the items, their alignment with curriculum objectives, and their ability to capture students' conceptual understanding of trigonometric ratios. Based on the expert's feedback, several revisions were made to improve the wording and clarity of the questions before the instrument was administered. The third phase consisted of classroom observations of six lessons on ratios and trigonometry. Each lesson lasted approximately 90 minutes and was documented through field notes and audio recordings. The observations focused on identifying the taught knowledge, including the conceptual explanations provided by the teacher, instructional strategies, representations, and classroom interactions. The final phase examined students' learned knowledge using the Ratios Trigonometry Test (RTT). The RTT consisted of fourteen items designed to measure students' conceptual understanding

of trigonometric ratios, relationships among triangle sides and angles, and their ability to apply these concepts in contextual situations.

Content validity of the instruments was ensured through alignment with the curriculum indicators and expert validation. The Readiness Test (RT) was reviewed by a mathematics education expert to ensure clarity, relevance to curriculum objectives, and appropriateness for capturing students' conceptual understanding. Reliability was supported through pilot testing and inter-coder agreement. The Readiness Test (RT) achieved an inter-coder agreement of 93.10%, while the Ratios Trigonometry Test (RTT) achieved 88.74%, indicating satisfactory reliability for qualitative interpretation of student responses.

A document analysis was conducted on national curriculum materials to characterize the knowledge to be taught. This analysis focused on Basic Competencies 3.7 and 4.7 and their corresponding indicators related to trigonometric ratios in right triangles (Kemendikbud, 2016). A summary of these competencies and indicators is presented in Table 1.

Table 1. Basic Competencies and Indicators for Trigonometric Ratios in the Curriculum (Kemendikbud, 2016)

Basic Competency	Indicators
3.7 Explain trigonometric ratios (sine, cosine, tangent, cotangent, secant, and cosecant) in right triangles	3.7.1 Determine the lengths of the sides of a right triangle using the Pythagorean theorem. 3.7.2 Determine the front side, side side and hypotenuse for an acute angle (α) in a right triangle 3.7.3 Explain trigonometric ratios in right triangles 3.7.4 Determine the trigonometric ratio values in a right triangle
4.7 Solve contextual problems related to trigonometric ratios	4.7.1 Create a mathematical model of problems related to trigonometric ratios in right triangles 4.7.2 Solve trigonometric ratio problems by measuring the height of a tower

Data analysis was conducted in several stages to examine the relationships among the knowledge to be taught, the taught knowledge, and the learned knowledge. First, a document analysis was carried out on national curriculum materials to characterize the knowledge to be taught. This analysis focused on Basic Competencies 3.7 and 4.7 and their corresponding indicators related to trigonometric ratios in right triangles (Kemendikbud, 2016). Curriculum documents were examined to identify the intended conceptual structures of trigonometric ratios within the curriculum. In parallel, teacher interview transcripts were analyzed to identify instructional intentions and the ways in which the teacher interpreted and organized the curricular content for classroom instruction. Based on these analyses, a conceptual network was constructed to represent the organization of curricular and instructional knowledge.

Subsequently, qualitative content analysis was applied to classroom observation

data and interview transcripts to analyze the taught knowledge enacted during instruction. The analysis focused on identifying the conceptual explanations provided by the teacher, the instructional tools used during teaching, the pedagogical strategies employed in the classroom, and the instructional language used to communicate trigonometric concepts. Through an inductive coding process, patterns in teaching practices and representations of trigonometric ratios were identified and interpreted within the instructional context. Students' learned knowledge was analyzed through a descriptive analysis of their responses to the Readiness Test (RT) and the Ratios Trigonometry Test (RTT). The RT results were used to characterize students' prior knowledge related to ratios and basic trigonometric concepts before instruction, while the RTT results were used to examine the knowledge students developed after the instructional sequence. Student responses were evaluated based on the learning objectives underlying each test item. Because each learning objective was measured through multiple items, the accuracy of students' understanding was determined by examining the consistency of their responses across items aligned with the same objective.

The final stage of the analysis focused on examining the relationships among the three forms of knowledge: the knowledge to be taught, the taught knowledge, and the learned knowledge. Differences and similarities among these knowledge structures were identified by comparing the conceptual networks derived from curriculum documents, classroom observations, teacher interviews, and student test results. These findings were interpreted within the framework of didactic transposition, enabling an analysis of how knowledge was transformed from its prescribed curricular form to its enacted instructional representation and ultimately to students' conceptual understanding. Although this study adopted a qualitative case study design, simple descriptive statistics such as frequencies and percentages were used to support the qualitative interpretation of students' responses. These numerical summaries were not intended for statistical generalization but rather to provide a clearer description of the observed patterns in students' understanding.

Results and Discussions

The Knowledge to be Taught

The identification of the knowledge to be taught was derived from an analysis of the official mathematics curriculum and a preliminary interview with the teacher. The teacher explained that the curriculum and textbook served as the primary references for planning instruction. Therefore, the teacher first examined the curricular objectives related to trigonometric ratios in right triangles and then described the intended structure of the instructional content. Based on the curriculum indicators presented in Figure 1, the topic of trigonometric ratios is introduced through several interconnected competencies. These include determining the lengths of the sides of a right triangle using the Pythagorean theorem, identifying the opposite side, adjacent side, and hypotenuse relative to a given acute angle, explaining the concept of

trigonometric ratios, and determining the values of trigonometric ratios in right triangles. In addition, students are expected to solve contextual problems involving trigonometric ratios by constructing mathematical models. These indicators reveal that the curriculum emphasizes both conceptual understanding of trigonometric ratios and their application in problem-solving situations.

In accordance with these curricular objectives, the teacher planned the instructional sequence to begin with the geometric interpretation of sine, cosine, and tangent in the context of right triangles. These functions were introduced as ratios between the sides of a right triangle relative to a given acute angle. After students understood these fundamental ratios, the teacher intended to extend the discussion to the reciprocal trigonometric ratios, namely cosecant, secant, and cotangent. The learning activities would then involve calculating trigonometric values, determining unknown sides and angles of right triangles, and applying the Pythagorean theorem to support these calculations. Finally, students would engage in contextual problem-solving tasks that require them to construct mathematical models and apply trigonometric reasoning. The analysis indicates that the intended knowledge domain centers on the geometric structure of the right triangle and the ratio relationships between its sides. These relationships serve as the conceptual foundation for defining the trigonometric functions. Furthermore, the teacher planned to incorporate classification and explanation activities to help students connect geometric representations with symbolic trigonometric expressions and their practical applications.

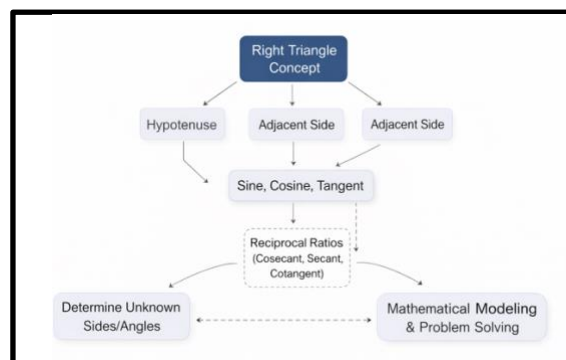


Figure 2. The network of the knowledge to be taught.

The conceptual organization of this knowledge is illustrated in Figure 2. The diagram shows that the concept of the right triangle functions as the geometric basis for introducing trigonometric ratios. Elements such as the hypotenuse, the side opposite the angle, and the adjacent side provide the structural components for defining the ratios sine, cosine, and tangent, as well as their reciprocal forms. Once these ratios are established, they are connected to further mathematical activities, particularly determining unknown sides and angles and solving contextual problems. The arrows in the diagram represent the progression of ideas from geometric foundations to the formal definition of trigonometric ratios and their application in mathematical modeling and problem solving.

The Taught Knowledge

The taught knowledge was identified through an analysis of classroom observations and teacher interviews, with particular attention to instructional resources, tools, pedagogical strategies, and the language employed by the teacher, as these elements shape the knowledge enacted in classroom practice (Isabel & Barbosa, 2023). In addition to the prescribed curriculum and textbook, the teacher utilized supplementary materials, including mathematics education references (John A. Van de Walle, 2015) and other practice-oriented sources. The teacher justified this decision by noting the limited definitions and examples provided in the textbook, emphasizing the need for additional conceptual and procedural support.

Instructional activities were primarily supported by non-digital tools, including an interactive board and concrete materials, such as paper, rulers, squared paper, and protractors. The teacher expressed reservations regarding the use of software such as GeoGebra, citing students' limited technological familiarity and the perceived mismatch between digital tools and students' learning habits. Consequently, classroom tasks relied mainly on paper-based constructions and measurements to facilitate conceptual understanding and to allow students to directly observe geometric relationships. The instructional approach reflected a student-centered orientation that emphasized experiential and exploratory learning. Students initially engaged in measurement activities, such as comparing body height and shadow length to investigate relationships within right triangles. These activities supported the development of proportional reasoning and served as an entry point to discussions about triangle congruence and similarity. Subsequent tasks connected these geometric relationships to the Pythagorean theorem and ratio reasoning, which later provided the foundation for introducing trigonometric ratios.

Evidence of students' conceptual understanding was also observed through interviews conducted during the lessons. The following exchange illustrates students' reasoning about congruent right triangles and their interpretation of trigonometric relationships as ratios between sides of a triangle.

- S1 : Congruent triangles are two triangles that have exactly the same side lengths and angle measures.
- S2 : Yes, they are identical in shape and size. No matter how you move, flip, or rotate them, they will perfectly match up.
- R : How can you determine if two right triangles are congruent?
- S2 : Because in a right triangle, when you know the lengths of the hypotenuse and one leg, you can find the other leg using the Pythagorean theorem. So those side lengths determine the whole triangle.
- R : Give me an example of congruent right triangles and explain how you know.
- S1 : Okay, let's say I have one right triangle with sides 5 inches, 12 inches, and 13 inches. And another triangle with sides 5 inches, 12 inches, and 13 inches. Since the hypotenuse of 13 and a leg of 12 are the same for both, the triangles are congruent by the hypotenuse-leg shortcut.
- R : When might knowing about congruent right triangles be useful?
- S2 : In geometry proofs involving right triangles. If you can show two are congruent, you know all the other parts must be equal too.

- R : When you learned trigonometry in class, how did you understand sine, cosine, and tangent?
- S1 : We learned them as comparisons between the sides of a right triangle. For example, sine is the opposite side divided by the hypotenuse.
- S2 : Yes, we usually look at the triangle and compare the sides around the angle. If we know two sides, we can calculate the ratio.
- R : What does the value of sine or cosine represent for you?
- S2 : It shows the relationship between the sides of the triangle. For example, sine tells us how long the opposite side is compared to the hypotenuse.

The interviews with the two students indicate a strong conceptual understanding of right triangle congruence. Both students correctly defined congruent triangles as figures with equal corresponding side lengths and angle measures, satisfying conditions such as Side-Side-Side (SSS), Angle-Side-Angle (ASA), or, specifically for right triangles, the Hypotenuse–Leg (HL) criterion. Student 1 explicitly stated that two right triangles are congruent if they share an equal hypotenuse and one corresponding leg. Student 2 justified this statement using the Pythagorean theorem, explaining that the hypotenuse and one leg uniquely determine the remaining leg. In addition, the interview responses revealed that students interpreted trigonometric ideas primarily as comparisons between the sides of a right triangle, such as the relationship between the opposite side, adjacent side, and hypotenuse. This suggests that students were able to work mathematically with trigonometric ratios in problem-solving situations, particularly when applying formulas to calculate unknown sides or angles. However, their explanations indicate that trigonometry was mainly understood as a set of side ratios within a triangle, rather than as functions that relate angle measures to numerical values.

During instruction, the teacher’s language played a central role in shaping the taught knowledge. She consistently employed formal mathematical terminology, emphasizing conceptual precision. Congruence was defined as invariance in shape and size independent of orientation, while angles were discussed not only quantitatively but also structurally through classifications and relational roles within triangles. Trigonometric ratios were introduced with explicit mathematical definitions:

$$\sin \alpha = \frac{\text{opposite}}{\text{hypotenusa}}, \quad \cos \alpha = \frac{\text{adjacent}}{\text{hypotenusa}}, \quad \tan \alpha = \frac{\text{opposite}}{\text{adjacent}}$$

The consistent use of formal mathematical discourse supported conceptual clarity, modeled mathematical rigor, and situated terminology within meaningful problem-solving contexts. In the initial lessons, students examined the structural properties of right triangles, recognizing the perpendicular relationship between the legs and the invariant right angle. Through measurement activities, students explored relationships among the sides and initially focused on special right triangles (30°–60°–90°, 45°–45°–90°). As additional triangles were analyzed, students observed that the ratios between sides varied systematically with the measure of the acute angle.

The teacher formalized these observations by introducing relational terms such as “opposite,” “adjacent,” and “hypotenuse,” and by explaining that trigonometric ratios remain positive for acute angles in the first quadrant. Subsequent lessons emphasized relationships among trigonometric ratios. Through guided discussions, students recognized the fundamental trigonometric identity $\sin^2\alpha + \cos^2\alpha = 1$, derived from $a^2 + b^2 = c^2$.

Students also examined limiting behavior as $\theta \rightarrow 0^\circ$ and $\theta \rightarrow 90^\circ$. Although exact values beyond special angles were not required, students developed the key understanding that trigonometric ratios depend solely on angle measure, not triangle size. This invariance was reinforced through physical models, where students verified that ratios such as $\frac{\text{opposite}}{\text{hypotenuse}}$ remained constant across similar triangles. Nevertheless, classroom activities and tasks largely emphasized the application of formulas and ratios in right triangles, which allowed students to successfully perform calculations but did not strongly emphasize the interpretation of trigonometric ratios as functions of an angle.

Building on these conceptual foundations, students applied trigonometric ratios to determine unknown sides and angles in right triangles. Instruction then extended to contextual problem-solving tasks involving heights and distances. Further lessons introduced the concepts of angles of elevation and angles of depression, supported by measurement tools such as clinometers. Through these practical projects, students connected abstract mathematical relationships to real-world applications, reinforcing both procedural competence and conceptual understanding. However, the tasks primarily involved substituting values into trigonometric formulas, which reinforced the interpretation of trigonometry as a set of ratios used to solve right triangle problems rather than as a broader functional relationship between angles and real numbers. The conceptual organization of the knowledge enacted during instruction, including the relationships among geometric properties, trigonometric ratios, and their applications, is illustrated in Figure 3.

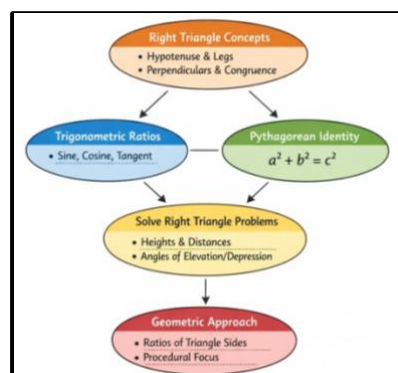


Figure 3. The network of the knowledge taught by the teacher

Figure 3 illustrates the conceptual network underlying the taught knowledge. The network begins with the structural understanding of right triangles, including the

identification of the hypotenuse and legs, as well as perpendicular relationships and triangle congruence. From these geometric foundations, students develop an understanding of ratios between triangle sides, leading to the formal definitions of sine, cosine, and tangent, which are connected to the Pythagorean identity and applied to determine unknown sides and angles in contextual problems involving heights and distances. While this instructional progression enabled students to successfully apply trigonometric ratios in solving right triangle problems, the emphasis remained largely on trigonometry as geometric ratios rather than as functions of angles. Consequently, students' understanding tended to remain within a geometric framework, which may limit their ability to transition from geometric reasoning to the analytic interpretation of trigonometric functions. As emphasized by Weber (2005), understanding trigonometric ratios as functions of angles rather than merely as relationships between sides is essential for students' long-term mathematical development.

The Learned Knowledge

The students' responses to the RTT indicate the knowledge they developed after instruction. To analyze their learning progression, the results of the initial and final tests on trigonometric ratios were compared, focusing on triangle congruence and the comparison of sine, cosine, and tangent. Table 2 presents the frequency and percentage of correct answers in both tests, showing the extent of students' conceptual development.

Table 2. The results of the analyses of the test related to on congruence of right triangles and comparison of trigonometric ratios

Concept	RT f	%	RTT f	%
Congruence of Right Triangles by HL (Hypotenuse-Leg)	2	28.6	6	85.7
Congruence of Right Triangles by SSS (Side-Side-Side)	1	14.3	5	71.4
Comparison of sine values for different angles	3	42.9	7	100
Comparison of cosine values for different angles	2	28.6	6	85.7
Comparison of tangent values for different angles	2	28.6	5	71.4
Finding unknown angles and unknown sides	1	14.3	4	57.1
Measuring heights and measuring distances	1	14.3	5	71.4

The data in Table 2 show noticeable improvements in students' performance on tasks related to right triangle congruence and the comparison of trigonometric ratios. The percentage of students correctly identifying congruent right triangles using the Hypotenuse–Leg criterion increased from the initial test to the final test. Similarly, all students were able to correctly compare sine values after instruction. Improvements were also observed in students' ability to compare cosine and tangent values and to apply trigonometric ratios in contextual problems such as measuring heights and

distances. These results indicate that the instructional activities effectively supported students' ability to apply trigonometric ratios and solve right triangle problems. However, a closer examination of students' responses suggests that this improvement was primarily related to procedural and ratio-based reasoning. Students were generally successful when tasks required identifying triangle sides, substituting values into trigonometric formulas, or comparing ratios based on angle size. Nevertheless, their explanations rarely reflected an interpretation of trigonometric ratios as functions of angles. Instead, most students described sine, cosine, and tangent as relationships between the sides of a right triangle. This pattern indicates that while students improved their ability to use trigonometric ratios in problem solving, their understanding remained largely within a geometric ratio framework, which may limit their transition toward the analytic perspective of trigonometric functions.

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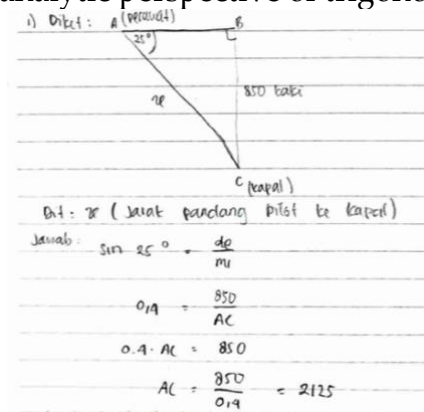


Figure 4. Student's solution to the trigonometric problem in

Figure 4 illustrates a student's written solution to a trigonometric problem, showing how a real-world situation was translated into a mathematical representation using trigonometric ratios. In solving the task, the student interpreted the angle of depression as the angle between the pilot's line of sight and the horizontal line and represented the situation as a right triangle. By identifying the aircraft's altitude as one side and the line of sight as the hypotenuse, the student applied the sine ratio to determine the unknown distance. This example reflects students' ability to model contextual situations mathematically and apply trigonometric ratios in problem-solving. Consistent with the data in Table 2, students' performance improved from the

initial test to the final test, with more students correctly identifying congruent right triangles using the Hypotenuse–Leg and Side–Side–Side criteria and successfully comparing sine, cosine, and tangent values. Students also demonstrated better use of the Pythagorean relationship and the connections among trigonometric ratios. However, the solutions largely relied on identifying triangle sides and substituting values into formulas, suggesting that students’ understanding remained primarily within a geometric ratio framework rather than a functional interpretation of trigonometric relationships.

The Relationship among the Knowledge to Be Taught, the Taught Knowledge, and the Learned Knowledge

The comparison among the knowledge to be taught, the taught knowledge, and the learned knowledge reveals a partial misalignment in the didactic transposition process. As shown in Table 2, students’ performance improved from the initial test to the final test, indicating that many students became more capable of identifying triangle relationships and applying trigonometric ratios in problem-solving tasks. However, a closer analysis of students’ responses suggests that this improvement was mainly related to procedural competence, such as recognizing triangle sides and substituting values into trigonometric formulas. Many students continued to interpret sine, cosine, and tangent primarily as ratios between the sides of a right triangle rather than as functions of an angle. Thus, although the teacher attempted to enrich the material beyond the prescribed curriculum, this expansion did not fully bridge the gap between the intended knowledge, the knowledge taught in the classroom, and the knowledge ultimately constructed by students. These findings highlight the complex and sometimes misaligned relationship among curricular knowledge, enacted classroom knowledge, and students’ constructed knowledge in mathematics learning. This dynamic can be more clearly understood through the framework of didactic transposition, which explains how knowledge is transformed across different educational levels.

Figure 5 presents the phases of didactic transposition identified in this study, adapted from the theoretical model of didactic transposition.

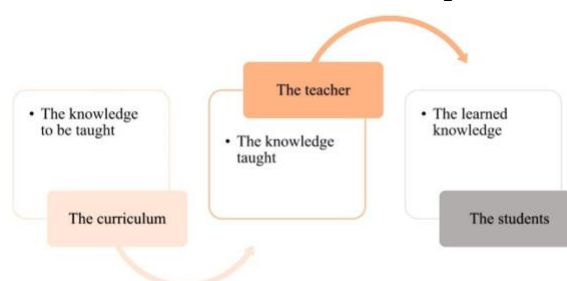


Figure 5. The didactic transposition phases in the study. Adapted from ‘Twenty-five Years of the Didactic Transposition’ by Bosch & Gascón (2006)

As a result, it can be argued that transpositions occurred from the knowledge to be taught to the learned knowledge within the internal didactic transposition process.

Although the teacher covered the content prescribed in the curriculum and even expanded it through instructional decisions, the knowledge ultimately learned by students remained limited to the basic properties of trigonometric ratios. In other words, while the taught knowledge extended beyond the prescribed knowledge, students did not fully appropriate this expansion. Their understanding did not encompass the broader conceptual explanations introduced during instruction, but instead focused mainly on procedural aspects of ratio relationships. Consequently, the learned knowledge was not fully aligned with the taught knowledge, leading to conceptual conflict between interpreting trigonometric ratios as relationships between sides and understanding them as functions of angles.

In the teaching of ratios and trigonometry in Indonesia, three forms of knowledge interact within the internal didactic transposition process: the knowledge to be taught, the taught knowledge, and the learned knowledge (Chevallard, 1991). The knowledge to be taught is prescribed by curriculum authorities representing the noosphere (Bosch & Gascón, 2006) and teachers have limited autonomy beyond the subject and thematic levels of didactic co-determination (Strømskag & Chevallard, 2024; Winsløw, 2011). In this study, although the teacher largely followed the prescribed curriculum, she expanded the content using external resources. This expansion slightly shifted the intended scope of instruction and influenced how knowledge was transformed in the classroom. The findings reveal that enriching content does not automatically lead to deeper understanding. Despite the teacher's effort to broaden and contextualize trigonometric concepts, students' understanding remained limited and in some cases regressed. This suggests limitations in pedagogical content knowledge and curriculum knowledge (Ball et al., 2008; Shluman, 1986) which are crucial for effective didactic transposition (Chevallard, 1991). While the teacher created exploratory activities and used visual representations to support reasoning (Watson, 2009), students struggled to transfer concepts to new contexts, indicating the presence of epistemological barriers that often arise when students attempt to connect previously learned concepts with new mathematical situations (Putranto, 2025). Students' misconceptions were consistent with findings reported in previous studies, including incorrect generalizations about triangles (Jesiek et al., 2014) and weak conceptual understanding of trigonometric relationships (Jesiek et al., 2014; Weber, 2005). These difficulties reflect broader conceptual challenges and epistemological barriers that often arise when students attempt to connect previously learned knowledge with new mathematical contexts (Mitchelmore & White, 2000; Putranto, 2025). Such learning obstacles may also be influenced by instructional practices, as similar challenges in supporting students' conceptual understanding have been documented among novice teachers (Essuman & Wilmot, 2024).

These results highlight the importance of strengthening professional competencies related to didactic transposition and pedagogical content knowledge. As

Chevallard, (1991) notes, one transposition process becomes the foundation for the next, meaning that unresolved misconceptions may hinder future learning. Therefore, curriculum documents should provide clearer methodological guidance, and professional development programs should include structured observation and feedback. Integrating visual models and manipulatives (Lithner, 2000), as well as considering the interdisciplinary nature of ratios and trigonometry within the Indonesian curriculum (Kemendikbud, 2016), may further support coherent conceptual development.

Conclusions and Suggestions

The study highlights a clear discrepancy between curricular goals and classroom practice in the teaching of trigonometric ratios. Although the curriculum emphasizes conceptual and functional understanding of trigonometry, classroom instruction predominantly presents trigonometric ratios as fixed relationships between the sides of right triangles. As a result, students tend to interpret sine, cosine, and tangent merely as comparisons of side lengths rather than as functions that depend on angle measures. This procedural and geometric emphasis limits students' ability to develop functional reasoning and creates difficulties when they are required to connect geometric representations with broader analytic interpretations. Consequently, students may struggle to transfer their knowledge to unfamiliar contexts or to understand trigonometric relationships beyond specific triangles. These findings underscore the need for stronger instructional coherence in curriculum implementation, ensuring that teaching practices align with the curriculum's conceptual and functional objectives. Future research could further investigate instructional approaches that explicitly connect geometric interpretations of trigonometric ratios with their functional representations, particularly in supporting students' transition from right-triangle reasoning to the analytic treatment of trigonometric functions.

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