WORKING COLLABORATIVELY IN PROVING GEOMETRY PROBLEMS TO ENHANCE UNDERGRADUATE STUDENTS' COMMUNICATION OF MATHEMATICAL THINKING

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Abstract

Learning mathematics does not come from individual understanding but because of the collaboration of knowledge. It happens when they communicate with each other. Therefore, this study aims to determine the effect of Collaborative Problem-Solving (CPS) on students’ mathematical communication and analyze students' mathematical communication through CPS. Thirty students became research participants and were divided into 15 groups so that one group consisted of two people. In collaboration with their colleagues, students solved the tasks of proof problems in geometry in pairs. Quantitative and qualitative approaches were used in this study to achieve each of the two research objectives. A scoring rubric for the level of mathematical communication was developed to determine students' mathematical communication. Qualitative analysis was carried out through interviews and group answers. The group selected was the one that experienced a significant increase in mathematical communication, the highest standard deviation value, and good communication. The results showed a different level of student mathematical communication in each meeting when students collaboratively solved geometry proof problems. This study
also indicates that forming CPS groups is better when group members have equal mathematical abilities.

**Keywords:** collaboration; communication; mathematical thinking; problem-solving

**INTRODUCTION**

Mathematics learning is inseparable from communication. Students learn mathematics in the learning process, which includes language and communication. All kinds of communicative actions, such as talking, drawing, and giving signs or gestures, are considered the primary source of human interaction and meaningful exchange (Steinbring, 2015). Research shows a link between math and communication skills (Hornburg et al., 2018; Purpura & Ganley, 2014; Toll & Van Luit, 2014). Purpura & Ganley (2014) find non-mathematical factors, such as language, closely related to mathematics. It is reinforced by the results of research, which show that, in the communication process, students develop their mathematical thinking after they convey and receive messages (Casadiego et al., 2023). Therefore, learning mathematics will be successful if it allows students to convey mathematical thinking.

Communication is conveying information to produce meaning in interactions with people, objects, or practices (Planas & Pimm, 2023). The 'people' in question are teachers and students. 'Objects' can be computers, books, or manipulative media, while examples of 'practice' are writing, explaining, or arguing. Yusoff et al. (2022) define communication as exchanging information with others. Communication methods can be done orally and in writing. The emergence of understanding in communication does not occur directly and cannot be transferred from one person to another (Steinbring, 2015). Each communication participant must build his understanding. One message cannot be understood directly by itself. However, the message's recipient must integrate this single information into a possible coherent network or nexus in which the information gets meaning (Steinbring, 2015). It is what is called understanding. People who understand will form a network of knowledge they already have with new knowledge (Hiebert & Carpenter, 1992).

Mathematical communication has a different meaning from communication. Mathematical communication is the ability of students to write, discuss, and listen about mathematics; the ability to connect mathematical ideas in oral or written communication with pictures, graphics, and natural objects; the ability to relate real pictures, objects, and diagrams with mathematical ideas (Latif, 2017; Rohendi, 2012). The National Council of Teachers of Mathematics (NCTM) emphasizes the importance of communication in mathematics and
mathematics education. NCTM suggests that mathematics programs should enable students to (1) organize and consolidate their mathematical thinking through communication; (2) communicate mathematical thinking coherently and clearly to peers, teachers, and other people; (3) analyze and evaluate the mathematical thinking and strategies of others; and (4) using mathematical language to express mathematical ideas appropriately.

Mathematical communication can encourage students to explain how they get answers by describing their mathematical thinking processes (Fried & Amit, 2003). Many students can solve mathematics problems correctly but do not understand how to achieve them. Kostos & Shin (2010) found that students can solve problems correctly but only use calculation methods instead of mathematical concepts. The student's response does not show a mathematical thinking process but shows the student's ability to remember and use methods to solve mathematical problems to find the correct answer (Kostos & Shin, 2010). It shows that students communicate but not in mathematical communication. They only convey what they remember but do not convey their mathematical thoughts. The delivery of mathematical thinking characterizes mathematical communication.

Students' understanding of a mathematical concept can be communicated in various ways: in writing, orally, through pictorial representations, and with manipulatives (Kostos & Shin, 2010). Students who understand mathematical concepts will be able to solve math problems easily. Research shows evidence of the identification of communicative skills that are manifested by students when solving math problems (Parada Rico et al., 2023). Through communication, an explanation can be a tool for understanding (Duval, 1995). It is because explanations can provide one or more reasons to make facts, phenomena, or results understandable. NCTM (2000) stated that when students communicate the results of their mathematical thinking to others, they learn to be clear and convincing.

Parada Rico et al. (2023) explain that mathematical communication skills can only be achieved if discussion and confrontation of ideas between students are promoted. It can support the need to convince themselves and others of the truth of their statements. These conditions can be supported in the context of students solving problems in collaboration, which is called Collaborative Problem-Solving (CPS). In CPS, students are given a space for interaction to discuss and convey ideas, justifications, and arguments to solve math problems. Therefore, mathematical communication research will be interesting if studied in the context of CPS.
Previous research has mainly been done on early-grade students in examining mathematical communication (Cooke & Buchholz, 2005; Kostos & Shin, 2010; Steinbring, 2015; Yusoff et al., 2022). This is because, in the early grades, students encounter mathematical communication problems. However, communication problems appear not only in low-grade students; undergraduate students also have mathematical communication problems. Therefore, this study focuses on undergraduate student subjects. Based on observations in mathematics education classes at the Universitas Muhammadiyah Malang, researchers found that students needed learning that supported them in communicating their mathematical ideas. Undergraduate students struggle to convey mathematical thinking and correctly write their mathematical ideas.

Albano et al. (2023) explore how students communicate their ideas in mathematics assignments through written text. However, this study did not explore how students express their ideas orally. In line with Albano et al. (2023), research conducted by Risalah & Hodiyanto (2022) used written mathematical communication to support students' abilities in mathematical proving. Students experience difficulties in solving problems through proof (Risalah & Hodiyanto, 2022). Proof problems require advanced mathematical abilities that students should have. One of the proof problems is found in the Euclidean Geometry. Therefore, this study aims to analyze students' mathematical communication both orally and in writing through CPS in proving geometric problems.

There are two research problems, namely: 1) Is there any effect of CPS on students' mathematical communication in solving geometry-proof problems? 2) how does student mathematical communication through collaborative problem-solving in proving geometric problems?

THEORETICAL FRAMEWORK

Communication of Mathematical Thinking and Problem-Solving

Mathematical communication and communication are two different things. Communication is conveying information to others (Yusoff et al., 2022). In comparison, mathematical communication delivers mathematical thinking (Kostos & Shin, 2010). The fundamental difference between communication and mathematical communication lies in "mathematical thinking." Therefore, the initial explanation of this theoretical framework is about thinking mathematically.
Mathematical thinking is related to problem-solving. Mathematical thinking is a process of solving mathematical problems through entry, attack, and review processes that involve specializing, generalizing, conjecturing, and convincing activities. The problem-solving process is a framework described by Mason, Burton, and Stacey (Antequera-Barroso et al., 2022). Table 1 shows Mason, Burton, and Stacey's problem-solving phases.

<table>
<thead>
<tr>
<th>Phase</th>
<th>Process</th>
<th>Issue or Proposition</th>
<th>Statement</th>
</tr>
</thead>
<tbody>
<tr>
<td>Entry</td>
<td>Specializing</td>
<td>What do I know? What do I want? What can I introduce?</td>
<td>STUCK!</td>
</tr>
<tr>
<td>Attack</td>
<td></td>
<td>Conjecturing Try Check and feel doubt But why?</td>
<td></td>
</tr>
<tr>
<td>Reviews</td>
<td>Generalizing</td>
<td>Check decision Reflect on key ideas and key moments Generalizing to a wider context</td>
<td>AHA!</td>
</tr>
</tbody>
</table>

The problem-solving phase by Muson, Burton, and Stacey (1992) introduces the concept of STUCK! and AHA! The concept relates to how to deal with problem-solving. When in the STUCK! position, many students feel frustrated and lack the motivation to move forward. At this stage, emotional control becomes an important role. Students who can manage confusion and frustration into positive emotions such as enthusiasm, motivation, and challenges will help solve problems. When students have successfully solved the problem, that stage is called AHA!

Furthermore, mathematical thinking is the ability to think and make decisions (Isoda, 2007). There are three categories of mathematical thinking, namely 1) mathematical thinking related to mathematical methods, 2) mathematical thinking related to mathematical content (ideas), and 3) mathematical thinking related to mathematical attitudes. Mathematical attitude is the driving category of the other two categories (Isoda, 2007). Mathematical attitudes consist of reasonable, objective, clarity, and sophistication. The mathematical method consists of inductive, analogical, deductive, integrative thinking, development, abstraction, simplifying, generalizing, specializing, symbolizing, as well as quantification and schematization. Meanwhile, mathematical ideas relate to mathematical content.

Based on this explanation, students who can communicate their mathematical thinking are said to have mathematical communication skills. There are two methods of mathematical communication: orally and in writing. There are several indicators of mathematical
communication, according to some experts, including the ability to connect mathematical relations or ideas in written or oral communication with pictures, graphs, and real objects, the ability to connect real objects, objects, and diagrams into mathematical ideas, and the ability to follow math questions that are relevant to problem situation (Latif, 2017; Rohendi, 2012).

Other researchers created a conceptual framework for making mathematical communication assessments that provide descriptions of mathematical communication abilities at the highest level as follows: provide complete answers with clear and unambiguous explanations and/or descriptions; may include appropriate and complete diagrams; communicate effectively to the identified audience; present strong supporting arguments that are reasonable and complete; may include examples and counter-examples (Cai et al., 1996). In line with Cai et al. (1996), which creates a level of mathematical communication, Lim & Pugalee (2004) make four levels of three criteria of mathematical communication. The three criteria for mathematical communication include 1) using a clear presentation, 2) using mathematical language, vocabulary, and symbols, and 3) selecting an algorithm and demonstrating computational abilities using the algorithm.

Based on the explanations of some of these experts, this study offers indicators of mathematical communication that are used as a basis for quantitative assessment of mathematical communication skills. This developed indicator simplifies the indicators initiated by (NCTM (2000), Cai et al., 1996; Lim & Pugalee, 2004) and emphasizes that mathematical communication is characterized by exposure to mathematical thinking. Therefore, mathematical communication criteria are based on three categories of mathematical thinking by Isoda (2007). Furthermore, the description of each criterion is adapted to the material raised in this study, namely the problem of proving type geometry. This study indicates mathematical communication skills, as written in Table 2.

<table>
<thead>
<tr>
<th>Criteria</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>The explanation of mathematical thinking is related to the mathematical method.</td>
<td>The explanation is shown using a systematic, complete, and correct geometry problem-proving algorithm based on postulates, theorems, and corollaries on geometry, either orally or in writing.</td>
</tr>
<tr>
<td>The explanation of mathematical thinking is related to mathematical ideas</td>
<td>Explanations are shown by correctly using terms, symbols, operations, and relations in geometric material, orally or in writing.</td>
</tr>
<tr>
<td>The explanation of mathematical thinking related to mathematical attitudes</td>
<td>The explanation is shown by providing reasonable and clear justification for the results of evidence that are carried out either orally or in writing.</td>
</tr>
</tbody>
</table>
Collaborative Problem-Solving (CPS)

CPS consists of two constructs, namely problem-solving and collaboration. Based on these two constructions, CPS has two aspects, namely cognitive aspects (problem-solving) and social aspects (collaboration) (Fiore et al., 2017; Pöysä-Tarhonen et al., 2018). Problem-solving occurs when a person does not know the routine procedures for completing the task (Schoenfeld, 2013). Meanwhile, collaboration occurs when more than one person does a task together (Roschelle & Teasley, 1995). Collaboration is characterized by being together from the beginning to the end of the problem-solving process. It is what distinguishes collaborative and cooperative. Cooperative occurs when there is a division of labor among group members to solve problems. In this study, CPS was carried out in the context of students completing assignments to prove geometric problems. Students work in pairs to collaborate to solve the problem. The selection of two people in one group considers the interaction between one student and his partner will be maximized. They will discuss with their partner because they are the only two who can solve problems together.

RESEARCH METHOD
Research Approach

This study focuses on how CPS can influence students' mathematical communication in proving geometric problems. In addition, this study also analyzes students' mathematical communication through CPS when solving problems proving geometry material. The mixed method was used in this study in which the effect of CPS on mathematical communication was examined using a quantitative approach as the first step. Then, mathematical communication in the context of CPS is analyzed using a qualitative approach as the second step.

Participants and Research Setting

The population of this study were 50 students. They were a second-semester students of the Mathematics Education Study Program at the Universitas Muhammadiyah Malang who were taking the Euclidean Geometry course. The number of sample is 30 students chosen by random sampling who are formed in groups of 2 people. Therefore, in this study, there were 15 groups studied. The researcher also acts as a lecturer teaching Euclid's Geometry course. The assignments given to students focused on congruence and similar triangles. Three lecture meetings were held to discuss these two materials. At the end of each lecture, students are given assignments to work on in collaboration with their respective groups.
Data collection

This research uses data collection methods as an assessment of the results of student group assignments and group interviews. There are three tasks given at the end of each lecture meeting. The task that was given must satisfy the criteria that can be done in the context of CPS. The criteria of the task were proof problems, open-ended problems, do not use a dynamic geometry environment, and geometry problems (Jamil, Siswono, Setianingsih, et al., 2023). The following list shows the three tasks assigned to student groups.

- First Meeting Task
  Look at the picture below. Given $\angle VRS \cong \angle TSR$ and $\overline{RV} \cong \overline{TS}$. Prove that $\triangle RST \cong \triangle SRV$.

- Second Meeting Task
  Look at the picture on the side. Given $\overline{PN}$ bisector $\overline{MQ}$. $\angle M$ and $\angle Q$ are right angles. Prove that $\triangle PQR \cong \triangle NMR$.

- Third Meeting Task
  Look at the picture on the side. Noted that $\overline{AB} \parallel \overline{DF}$ and $\overline{BD} \parallel \overline{FG}$. Prove that $\triangle ABC \sim \triangle EFG$.

Interviews were conducted with a group representative that experienced a significant increase in mathematical communication. In addition, the group was selected based on the highest standard deviation score. It shows a high variation in scores, thereby enriching the research findings. This interview data is used to complement the results of the analysis of students' mathematical communication skills orally in addition to the results of their written assignments.

Data analysis

There were two data analysis procedures in this study: quantitative and qualitative. The first procedure was that the data was analyzed quantitatively, namely 1) quantitative holistic scoring procedure and 2) quasi-experiment design. In the quantitative holistic scoring procedure, each student assignment result is given a score ranging from 1 to 4 based on criteria
at the mathematical communication level. These criteria are derived from the indicators of mathematical communication skills shown in Table 2. The creation of a scoring rubric for mathematical communication was inspired by research (Lim & Pugalee, 2004). Table 3 shows the scoring rubric based on the level of mathematical communication used in this study. Based on the scoring rubric, group work results identified the level of mathematical communication at each meeting.

In the quasi-experiment design, the effect of CPS on students' mathematical communication was defined by a difference in mathematical communication level mean and an increase in the mean in each meeting. By giving a treatment of CPS in the three meetings, we have the score of mathematics communication level in three meetings of each group. Then, the data was examined using a paired sample t-test to answer the following research hypothesis.

H0: There is no difference in mathematical communication level in CPS for the first, second, and third meetings.

H1: There is a difference in mathematical communication levels in CPS for the first, second, and third meetings.

In answering the hypothesis, the paired sample t-test was done by comparing the data of the first and second meetings, the second and third meetings, and the first and third meetings. It was done to justify the validity of the research findings.

<table>
<thead>
<tr>
<th>Criteria</th>
<th>Level 1</th>
<th>Level 2</th>
<th>Level 3</th>
<th>Level 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>The explanation of mathematical thinking is related to the mathematical method.</td>
<td>The explanations are shown using unsystematic, incomplete, and incorrect geometry problem-proving algorithms based on postulates, theorems, or corollaries on geometry, either orally or in writing.</td>
<td>The explanations are shown using an unsystematic, less complete, and less correct geometry-proving algorithm based on postulates, theorems, and corollaries on geometry, either orally or in writing.</td>
<td>The explanations are shown using algorithms to prove geometry problems that are less systematic, incomplete, or incorrect based on postulates, theorems, or corollaries on geometry, either orally or in writing.</td>
<td>The explanations are shown using a systematic, complete, and correct geometry problem-proving algorithm based on postulates, theorems, and corollaries on geometry, either orally or in writing.</td>
</tr>
<tr>
<td>The explanation of mathematical thinking is related to mathematical ideas</td>
<td>The explanations use terms, symbols, operations, or relations inappropriate in geometric material, either orally or in writing.</td>
<td>The explanations are shown using terms, symbols, operations, and relations that are, to a lesser extent, appropriate to the geometry material, either orally or in writing.</td>
<td>The explanations are indicated by using terms, symbols, operations, and relations most appropriate to the geometry material, either orally or in writing.</td>
<td>The explanations are shown by correctly using terms, symbols, operations, and relations in geometric material, either orally or in writing.</td>
</tr>
<tr>
<td>Criteria</td>
<td>Level 1</td>
<td>Level 2</td>
<td>Level 3</td>
<td>Level 4</td>
</tr>
<tr>
<td>-------------------------------------------------------------------------</td>
<td>-------------------------------------------------------------------------</td>
<td>-------------------------------------------------------------------------</td>
<td>-------------------------------------------------------------------------</td>
<td>-------------------------------------------------------------------------</td>
</tr>
<tr>
<td>The explanation of mathematical thinking related to mathematical attitudes</td>
<td>The explanations are shown by providing unreasonable and unclear justifications for the proof results, either orally or in writing.</td>
<td>The explanations are indicated by the provision of reasons that make no sense and are not clear enough for the results of proof, either orally or in writing.</td>
<td>The explanations are shown by giving reasonable but unclear justifications for the results of proof that are carried out either orally or in writing.</td>
<td>The explanations are shown by providing reasonable and clear justification for the results of proof that are carried out either orally or in writing.</td>
</tr>
</tbody>
</table>

In the qualitative analysis, student assignments and interview results were not given a numerical score but were classified into specific categories according to the level of mathematical communication. This study used interactive data analysis model (Miles et al., 2014) that consist of three stages: data condensation, data display, drawing and verifying conclusions. The qualitative analysis procedure examines students' mathematical communication from two different perspectives. The two perspectives are the quality of written mathematical communication, as described in the rubric in Table 3, and the quality of oral mathematical communication, which meets clear criteria and makes it easier for listeners to understand how they complete their assignments.

RESULT AND DISCUSSION

Result

*The Influence of CPS on Student Mathematical Communication*

Groups comprised two students who were asked to sit close together in three lecture meetings. This arrangement of sitting close together has the goal that students are accustomed to discussing the material being studied with their pair, namely the congruence and similarity of triangles. Students are given assignments to collaborate with their colleagues at the end of each lecture.

Mathematical problems are divided into 'problem to find' and 'problem to prove' (Bell & Polya, 1945). 'Problem to find' is important in basic mathematics, while 'problem to prove' is used in advanced mathematics (Bell & Polya, 1945). In addition, 'problem to prove' can trigger students to discuss because it requires advanced mathematical skills. Problems that trigger difficulties to work on will trigger a person's desire to ask questions and exchange opinions with others. Therefore, CPS can be more appropriate in solving problems of the 'problem to prove' type, although CPS can still be used in solving problems of the 'problem to find' type.
Results of student assignments are given a score of one to four based on the level criteria described in Table 3. The scoring results are summarized in Table 4. There were 30 students as research participants, so there were 15 groups.

<table>
<thead>
<tr>
<th>Group</th>
<th>Mathematical Communication Level</th>
<th>Conclusion</th>
<th>Standard Deviation Score</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>First Meeting Tasks</td>
<td>Second Meeting Task</td>
<td>Third Meeting Task</td>
</tr>
<tr>
<td>Group 1</td>
<td>4</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>Group 2</td>
<td>4</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>Group 3</td>
<td>1</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>Group 4</td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>Group 5</td>
<td>3</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>Group 6</td>
<td>1</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>Group 7</td>
<td>3</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>Group 8</td>
<td>4</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>Group 9</td>
<td>4</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>Group 10</td>
<td>1</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>Group 11</td>
<td>1</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>Group 12</td>
<td>3</td>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>Group 13</td>
<td>1</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>Group 14</td>
<td>1</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>Group 15</td>
<td>2</td>
<td>2</td>
<td>4</td>
</tr>
</tbody>
</table>

Based on data from Table 4, the conclusion "increase" is given when the group experiences a consistent increase in the score at each meeting or is dominant at a high score of 3 or 4. Meanwhile, the conclusion "decrease" is given to groups that experience a consistent decrease in scores at each meeting or dominant is at a low score of 1 or 2. In Table 4, there are ten groups with an increased level of mathematical communication. It is equivalent to 67% of the group showing good changes in mathematical communication skills through CPS. Meanwhile, the five groups that got the mathematical communication score fell. Thus, it can be seen that 33% of the group showed changes in communication skills that are not good through CPS. CPS is effective in supporting students in developing their mathematical communication skills. An increase in mathematical communication level indicates it.

The data was examined using a z-test to determine the normality of the data. Based on the z-test that was shown in Table 5, all z-scores at the first, second, and third meetings show that $-1.96 < z \text{ score} < 1.96$. It means that the data is normal. Therefore, the t-test can be done.
To answer the research hypothesis, we examined whether there were differences in the level of mathematical communication between the first and second meetings, the second and third meetings, and the first and third meetings. Table 6 shows the result of the paired sample t-test. For pair 1 (first and second meeting), we can see that the significance score is 0.37. The significance score of pair 2 (second and third meeting) is 0.506. Pair 3 (first and third meeting) has a 0.642 significance score. Thus, all the significance scores are greater than 0.05. It can be concluded that the rejection of H0 is due to differences in mathematical communication levels between the meetings.

<table>
<thead>
<tr>
<th>Pair</th>
<th>First &amp; Second</th>
<th>Second &amp; Third</th>
<th>First &amp; Third</th>
</tr>
</thead>
<tbody>
<tr>
<td>N</td>
<td>Correlation</td>
<td>Sig.</td>
<td></td>
</tr>
<tr>
<td>Pair 1</td>
<td>First &amp; Second</td>
<td>15</td>
<td>0.246</td>
</tr>
<tr>
<td>Pair 2</td>
<td>Second &amp; Third</td>
<td>15</td>
<td>0.186</td>
</tr>
<tr>
<td>Pair 3</td>
<td>First &amp; Third</td>
<td>15</td>
<td>0.131</td>
</tr>
</tbody>
</table>

Further information about the differences can be seen in Table 7. We can see that the mean of the first meeting is 2.2667, the second meeting is 2.0667, and the third meeting is 2.9333. It means that there is a decrease in the mean in Pair 1. The decrease can see the difference in Pair 1 of 0.2 mean score. But for Pair 2 and Pair 3, there was an increase of mean, 0.8666 and 0.666, respectively.
Table 7. The result of Paired Samples Statistics

<table>
<thead>
<tr>
<th>Pair</th>
<th>First</th>
<th>Second</th>
<th>Third</th>
</tr>
</thead>
<tbody>
<tr>
<td>N</td>
<td>Mean</td>
<td>Std. Deviation</td>
<td>Std. Error Mean</td>
</tr>
<tr>
<td>15</td>
<td>2.2667</td>
<td>1.33452</td>
<td>0.34457</td>
</tr>
<tr>
<td>15</td>
<td>2.0667</td>
<td>0.59362</td>
<td>0.15327</td>
</tr>
<tr>
<td>15</td>
<td>2.9333</td>
<td>1.33452</td>
<td>0.34457</td>
</tr>
</tbody>
</table>

In addition, two groups consistently have a very good level of mathematical communication: a score of 3 and two scores of 4. The two groups are Group 1 and Group 2. Based on data on student GPA scores in the two groups, these students have the highest GPA scores in their class. Two of them have a perfect GPA of 4.00. Thus, it becomes natural if they obtain a high level of mathematical communication. In addition, collaboration between students becomes more effective than other groups because they already have a good basic knowledge and understanding to complete assignments.

Interesting findings were shown by Group 12, where at the first meeting, the results of their work included a good level of mathematical communication, namely 3. However, at the second meeting, their level of mathematical communication dropped to 1. The interview results showed that at the second meeting, their understanding of the question's intent was inadequate. This group consists of two students of the same gender, namely boys. The interview results also showed that they did not correct their written answers. Based on the GPA data, these two students have GPAs classified as sufficient and equivalent, namely 3.38 and 3.43. This group did not repeat the same mistake at the second meeting; the third meeting showed increased mathematical communication and the highest score.

Another interesting finding was shown by Group 10. This group was consistent with a low level of mathematical communication in all three lecture meetings. The results of the interviews indicated that they could not determine the appropriate postulates, theorems, or properties. Their work is not systematic, and the reasons written are not precise. Based on the GPA data, their scores are not equal. One of them has a very good GPA, namely 3.83, while his colleague has a GPA of 3.25. Differences in mathematical abilities based on GPA can indicate successful collaboration in CPS. The unequal mathematical ability indicates collaboration is not working effectively enough, which causes students' mathematical communication to be low.
Student Mathematical Communication Through CPS When Solving Problems of Proving Geometry Material

From the previous result, which showed differences in students' mathematical communication levels when they solved problems collaboratively, we can say that CPS affects students' mathematical communication levels. Furthermore, we analyzed written and oral mathematical communication from group work and group interviews, respectively. We found three groups that experienced a significant increase in mathematical communication and the highest standard deviation score: Group 3, Group 6, and Group 13 (see Table 4). However, we chose only Group 3 to describe the mathematical communication because the group has good communication. We believe the data from Group 3 can enrich the findings of this research.

In Figure 1, the group does not re-draw the shape given to the problem (see the First Meeting Task in the Research Methods section). Group 3 can identify the purpose of the question and what the purpose of the question is. It is shown by students writing down what is known and requested by the problem. However, this writing is not complete. Students write down what is asked simply by \( \triangle RST \cong \triangle SRV \). It should be written in complete form, i.e., prove that \( \triangle RST \cong \triangle SRV \). In the proof answers, Group 3 repeatedly wrote down what was known to the question, which showed an unsystematic indication. It is indicated by the words, "Given \( \angle VRS \cong \angle TSR \) and \( \overline{RV} \equiv T\overline{S} \) \n
**English Version:**

- Given: \( \angle VRS \cong \angle TSR \) and \( \overline{RV} \equiv T\overline{S} \)
- Asked: \( \triangle RST \cong \triangle SRV \)

Proof: Given \( \angle VRS \cong \angle TSR \) implies \( m\angle VRS = m\angle TSR \) that means \( \angle VRS \cong \angle TSR \). Given \( \overline{RV} \equiv T\overline{S} \), thus based on ASA Postulate, then \( \triangle RST \cong \triangle SRV \).

The interview results in Group 3 supported the researcher's analysis of students' mathematical communication skills after the first task. The researcher asked Group 3 to explain...
the results of their work orally. The oral explanation given by Group 3 was the same as what had been written down. After the researcher asked to pay attention again to what was missing in the answers that had been written, then they realized that there was an unsystematic nature in what they wrote. It shows that the use of mathematical terms and relations with students is a little appropriate. Therefore, the justification for the evidence is unclear and unreasonable. The following interview excerpt 1 shows the group's oral explanation of the answers to the first task. Interview Excerpt 1

Researcher: "Could you explain how you prove \( \triangle RST \cong \triangle SRV \)?"

Group 3: "In the given problem, the VRS angle is congruent with the TSR angle. It means that the VRS angle is the same as the TSR angle. Therefore, angle VRS is congruent with angle TSR. Also, given that the RV side is congruent with the TS side. Based on the ASA Postulate, it is proven that the RST triangle is congruent with the SRV triangle."

Researcher: "Are you sure your proof is systematic, complete, and correct?"

Group 3: "Apparently so"

Researcher: "Try to check the answers you have written. Is there something missing or enough?"

Group 3: "Oh yes. There is something strange. Why are our answers confusing? The angle VRS is known to be congruent to the angle TSR; the conclusion we write is also the same."

Researcher: "Are the ASA Postulates appropriate as your basis for proving what the question asks for?"

Group 3: "We have only shown one pair of sides. We think our proof is incomplete."

At the second meeting, there was an increase in Group 3's mathematical communication skills, although it was not significant. Figure 2 shows the work of group 3 on the second meeting task. In Figure 2, group 3 re-draws the known shapes in the problem. The group used the SAS Postulates (Sides-Angles-Sides) to answer this second task. Group 3 has shown two pairs of congruent sides and one pair of congruent angles of the two triangles being asked about. However, the angle indicated by Group 3 is not the angle flanked by the two indicated sides. The use of SAS can prove this second task, but group 3 has not been able to provide a pair of angles that are in accordance with the postulates. Writing symbols, terms, and relations shows that most are correct. However, the answers are less reasonable and unclear because of the
mismatch of the indicated angles. Therefore, the students' mathematical communication level can be categorized as level 2 in this second task. There is a one-level increase from the previous task. We have completed Figure 2 with a translation in English.

English Version:
Given: $\overline{PN}$ bisect $\overline{MQ}$
$\angle M$ and $\angle Q$ are right angles
Asked: $\triangle PQR \cong \triangle NMR$
Proof:
$\overline{PN}$ and $\overline{MQ}$ bisect each other at R. Therefore, $\overline{MR} \cong \overline{RQ}$ and $\overline{PR} \cong \overline{RN}$. Given that $\angle M$ and $\angle Q$ are right angles, $m\angle M = 90^\circ$ and $m\angle Q = 90^\circ$. Therefore, $m\angle M = m\angle Q$ that means $\angle PQR \cong \angle NMR$. Based on the SAS Postulate, then $\triangle PQR \cong \triangle NMR$.

Figure 2. The Work of Group 3 on The Second Meeting Task

Group 3 showed a significant increase in the third meeting task. Group 3 writes answers systematically and uses reasonable and correct justifications. The use of symbols and mathematical terms in the written answers is correct. However, there is one stage that they put together so that the work looks incomplete. However, that does not detract from the truth of their work. Figure 3 shows the work of group 3 on the third meeting task. Figure 3 is equipped with a translation in English. The group has been more careful in proving what the questions asked are based on the results of the interviews conducted by the researcher. The experience of working collaboratively in the context of CPS makes them more thorough and concerned about using the right algorithm in solving problems.
In Figure 3, the group did not use long and detailed justification sentences, but the evidence that was carried out was clear. Answers start with what they know from the problem and use the line AG as the transversal to prove two pairs of angles that are the same size. So, based on Corollary AA (Angles-Angles), it can be proven that the triangles ABC and DEF are congruent. Based on the clarity and accuracy of methods, content, and mathematical attitudes in group 3 work, it can be shown that the group's mathematical communication skills are included at level 4.

Table 8. The Difference of Mathematical Communication Skills in The First, Second, and Third Meeting of Group 3

<table>
<thead>
<tr>
<th></th>
<th>First Meeting</th>
<th>Second Meeting</th>
<th>Third Meeting</th>
</tr>
</thead>
<tbody>
<tr>
<td>The group does not</td>
<td>re-draw the shape given to the problem.</td>
<td>The group re-draws the known shapes in the problem.</td>
<td>The group re-draws the known shapes in the problem.</td>
</tr>
<tr>
<td>The group do not</td>
<td>write down completely what is known and what is asked.</td>
<td>The group write down symbols, terms, and relations that shows mostly correct.</td>
<td>The group writes answers systematically and uses reasonable and correct justifications.</td>
</tr>
<tr>
<td>The group repeatedly</td>
<td>wrote down what was known to the question, which showed an unsystematic indication.</td>
<td>The group’s answers are less reasonable and unclear.</td>
<td>The group did not use long and detailed justification sentences, but the evidence that was carried out was clear</td>
</tr>
<tr>
<td>The proof is</td>
<td>incomplete.</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>The group is</td>
<td>classified at Level 1.</td>
<td>The group classified at Level 2.</td>
<td>The group classified at Level 4.</td>
</tr>
</tbody>
</table>

English Version:
Because of $\overline{AB} \parallel \overline{DF}$ and $\overline{BD} \parallel \overline{FG}$ (given), then AG is a transversal. Therefore, $\angle BAC \cong \angle FGE$ (corresponding angles). Based on AA Corollary, proved that $\triangle ABC \sim \triangle EFG$. 

Figure 3. The work of Group 3 on The Third Meeting Task.
Discussion

The results of the study show that there is a difference in mathematical communication level when students work collaboratively in solving proof problems. Thus, the CPS has an effect on students' mathematical communications. Working collaboratively in a group can encourage students to exchange ideas and knowledge to solve problems. The exchange of mathematical ideas is manifested in mathematical communication. The delivery of mathematical thinking characterizes mathematical communication. As Isoda (2007) explained, mathematical thinking consists of three categories: mathematical ideas, mathematical methods, and mathematical attitudes. The exchange of mathematical ideas and methods and mathematical attitudes can appear in CPS. Someone with a positive attitude is more likely to be able to develop mathematical problem-solving skills (Xue et al., 2021; Zsoldos-Marchis, 2015). The results showed that students who learned to use collaborative methods in solving problems had statistically significant positive changes (Zsoldos-Marchis, 2015), as an example of student's beliefs about the usefulness of mathematics.

Collaborative learning currently has great potential in education because it can promote the construction of shared knowledge and develop students' abilities to learn to work in teams (Herrera-Pavo, 2021; Sulaiman & Shahrill, 2015). Interaction is important when students work collaboratively to solve problems (Barron, 2003; Jamil, Siswono, & Setianingsih, 2023). However, not all collaborations can run effectively. Molenaar et al. (2014) found that some students were known to ignore the contributions of their peers. At the same time, the main characteristic of CPS is the togetherness to achieve a common goal (Roschelle & Teasley, 1995). Neglecting the contributions of teammates can diminish the hallmarks of CPS. This study also found that there were groups with low levels of mathematical communication skills as a result of ineffective collaborative work. The difference in their mathematical abilities is significant enough that students with higher mathematical abilities seem to ignore the role of their colleagues. It impacts the group's work results, which is not optimal. This incident can be a second indication that CPS influences students' mathematical communication skills.

This research shows evidence that equality of mathematical ability also influences collaborative performance. Groups with equal mathematical abilities make collaborative work better. It can happen because students feel they need each other's teamwork, and none of their teammates underestimate each other. Damon & Phelps (1989) explained that CPS occurs when interactions occur in students with the same competency level to share their ideas to solve
challenging problems together. The results of this study do not declare that the formation of groups in CPS must be of equal ability, but this can help the effectiveness of the interactions that occur in them, especially in the context of mathematical communication. Students who communicate mathematically are students who can convey their mathematical thoughts. Mathematical thinking can be honed well when conveyed to colleagues who understand their thinking. It occurs when colleagues have equivalent mathematical knowledge. Groups with different abilities will be better suited to peer tutoring as defined by (Damon & Phelps, 1989). Peer tutoring allows team members with better mathematical abilities to help understand their teammates.

CONCLUSION

CPS affects students' mathematical communication levels. The problem of proving geometry, which is a challenging problem, gives space for students to express their mathematical thoughts to one another. The essence of mathematical communication is the delivery of mathematical thoughts, which in this study focuses on mathematical ideas, mathematical methods, and mathematical attitudes. In addition, this study also provides evidence that the habit of working collaboratively in the form of CPS impacts the development of students' mathematical communication skills. The findings show that equal mathematical ability in groups will help CPS interaction activities to be effective. It can be a reference for educators to organize groups with equal abilities to discuss with each other to solve problems. However, it would be interesting if further research could apply peer tutoring to teams that have members with significantly different mathematical abilities and see how their mathematical communication skills develop.

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